Unit 2
Real And Complex Numbers

EXERCISE 2.1

Q1. Identify which of the following are rational and irrational numbers.

(i) $\sqrt{3}$ **(ii)** $\frac{1}{6}$ **(iii)** π

(iv) $\frac{15}{2}$ **(v)** 7.25 **(vi)** $\sqrt{29}$

Solution:

Rational Numbers

All numbers of the form p/q where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q,

i.e., $Q = \left\{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1\right\}$

Irrational Numbers

The numbers which cannot be expressed as quotient of integers are called irrational numbers.

The set of irrational numbers is denoted by Q',

i.e., $Q' = \left\{\frac{x}{x} \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0\right\}$

For example, the numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$ and e are all irrational numbers. The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R,

i.e., $R = Q \cup Q'$

Here Q and Q' are both subset of R and $Q \cap Q' = \phi$

Rational Numbers	**ii, iv, v**
Irrational Numbers	**i, iii, vi**

Q2. Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$

Solution:

$$\begin{array}{r|l} & 0.68 \\ \hline 25 & 170 \\ & 150 \\ \hline & 200 \\ & 200 \\ \hline & 0 \end{array}$$

So, $\frac{17}{25} = 0.68$

(ii) $\frac{19}{4}$

Solution:

$$\begin{array}{r|l} & 4.75 \\ \hline 4 & 19 \\ & 16 \\ \hline & 30 \\ & 28 \\ \hline & 20 \\ & 20 \\ \hline & 0 \end{array}$$

So, $\frac{19}{4} = 4.75$

(iii) $\frac{57}{8}$

Solution:

$$\begin{array}{r|l} & 7.125 \\ \hline 8 & 57 \\ & 56 \\ \hline & 10 \\ & 08 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \\ \hline & 0 \end{array}$$

So, $\frac{57}{8} = 7.125$

(iv) $\frac{205}{18}$

Solution:

$$
\begin{array}{r|l}
 & 11.3889 \\
\hline
18 & 205 \\
 & 18 \\
\hline
 & 25 \\
 & 18 \\
\hline
 & 70 \\
 & 54 \\
\hline
 & 160 \\
 & 144 \\
\hline
 & 160 \\
 & 144 \\
\hline
 & 160
\end{array}
$$

So, $\frac{205}{18} = 11.3889$

(v) $\frac{5}{8}$

Solution:

$$
\begin{array}{r|l}
 & 0.625 \\
\hline
8 & 50 \\
 & 48 \\
\hline
 & 20 \\
 & 16 \\
\hline
 & 40 \\
 & 40 \\
\hline
 & 0
\end{array}
$$

So, $\frac{5}{8} = 0.625$

(vi) $\frac{25}{38}$

Solution:

$$
\begin{array}{r|l}
 & 0.65789 \\
\hline
38 & 250 \\
 & 228 \\
\hline
 & 220 \\
 & 190 \\
\hline
 & 300 \\
 & 266 \\
\hline
 & 340 \\
 & 304 \\
\hline
 & 360 \\
 & 342 \\
\hline
 & 18
\end{array}
$$

So, $\frac{25}{38}$ = 0.65789

Q3. Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ **is an irrational number.**

(ii) π **is an irrational number.**

(iii) $\frac{1}{9}$ **is a terminating fraction.**

(iv) $\frac{3}{4}$ **is a terminating fraction.**

(v) $\frac{4}{5}$ **is a recurring fraction.**

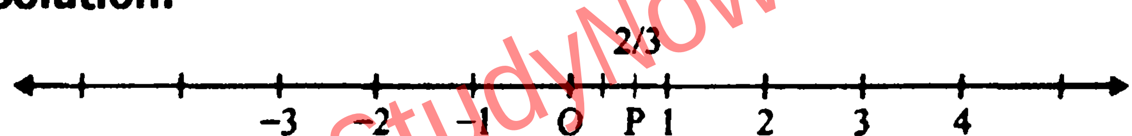
Solution:

(i) False **(ii)** True **(iii)** False
(iv) True **(v)** False

Q4. Represent the following numbers on the number line.

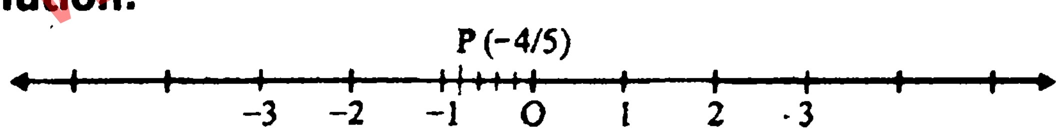
(i) $\frac{2}{3}$

Solution:



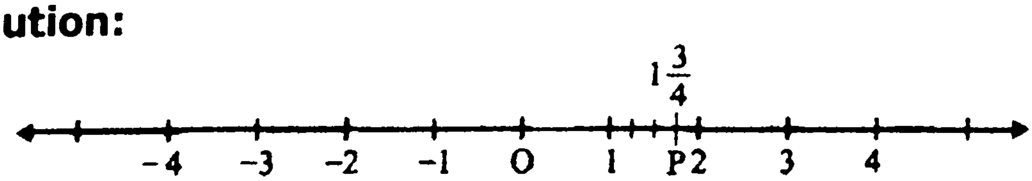
(ii) $-\frac{4}{5}$

Solution:



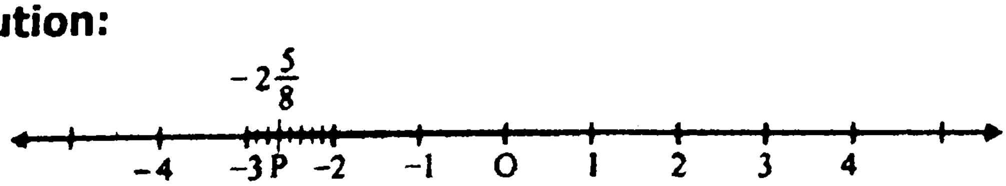
(iii) $1\frac{3}{4}$

Solution:



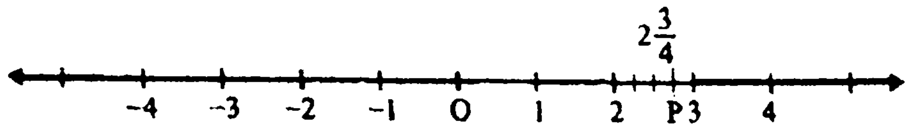
(iv) $-2\frac{5}{8}$

Solution:



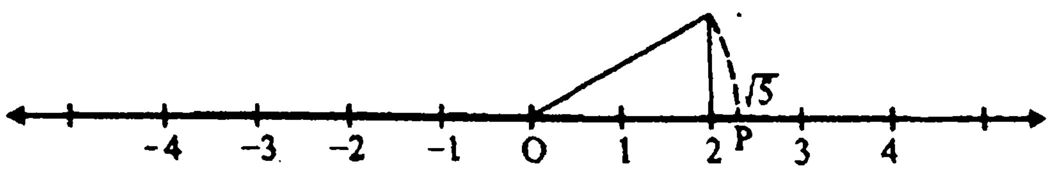
(v) $2\frac{3}{4}$

Solution:



(vi) $\sqrt{5}$

Solution:



Q5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Solution:

Sum of numbers = $\frac{3}{4}+\frac{5}{9} = \frac{27+20}{36} = \frac{47}{36}$

Rational number between $\frac{3}{4}$ and $\frac{5}{9}$ = $\frac{\left(\frac{47}{36}\right)}{2}$

$= \frac{47}{36}\times\frac{1}{2} = \frac{47}{72}$

Q6. Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integers and $q \neq 0$.

(i) $0.\overline{5}$ **(ii)** $0.\overline{13}$ **(iii)** $0.\overline{67}$

(i) $0.\overline{5}$

Solution:

Let $x = 0.\overline{5}$

or $x = 0.5555 \ldots\ldots$ .... **(i)**

Since we have only one digit i.e., 3 repeating indefinitely therefore multiplying both sides by 10

$10x = 5.555 \ldots\ldots$ .... **(ii)**

Subtracting (i) from (ii), we get

$10x - x = (5.555 \ldots\ldots) - (0.555 \ldots\ldots)$

$9x = 5.000$

Hence $x = \frac{5}{9}$

(ii) $0.\overline{13}$

Solution:

Let $x = 0.\overline{13}$

or $x = 0.131313 \ldots\ldots$ .... **(i)**

Since we have only two digit i.e., 13 repeating indefinitely therefore multiplying both sides by 100

$100x = 13.131313 \dots\dots$ .... **(ii)**

Subtracting (i) from (ii), we get

$100x - x = (13.1313 \dots\dots) - (0.1313 \dots\dots)$

$99x = 13.000$

Hence $x = \frac{13}{99}$

(iii) $\mathbf{0.\overline{67}}$

Solution:

Let $x = 0.\overline{67}$

or $x = 0.676767 \dots\dots$ .... **(i)**

Since we have only two digit i.e., 67 repeating indefinitely therefore multiplying both sides by 100

$100x = 67.676767 \dots\dots$ .... **(ii)**

Subtracting (i) from (ii), we get

$100x - x = (67.6767 \dots\dots) - (0.6767 \dots\dots)$

$99x = 67.000$

Hence $x = \frac{67}{99}$

EXERCISE 2.2

Q1. Identify the property used in the following

(i) $a + b = b + a$ **(ii)** $(ab)c - a(bc)$

(iii) $7 \times 1 = 7$ **(iv)** $x > y$ **or** $x = y$ **or** $x < y$

(v) $ab = ba$ **(vi)** $a + c = b + c \Rightarrow a = b$

(vii) $5 + (-5) = 0$ **(viii)** $7 \times \frac{1}{7} = 1$

(ix) $a > b \Rightarrow ac > bc \quad (c > 0)$

Solution:

(i) Commutative Property w.r.t Addition

(ii) Associative Property w.r.t. Multiplication

(iii) Multiplicative Identity

(iv) Trichotomy Property

(v) Commutative Property w.r.t. Multiplication

(vi) Cancellation Property of Addition

(vii) Additive Inverse

(viii) Multiplicative Inverse

(ix) Multiplicative Property

Since we have only two digit i.e., 13 repeating indefinitely therefore multiplying both sides by 100

$100x = 13.131313\ldots\ldots$ .... **(ii)**

Subtracting (i) from (ii), we get

$100x - x = (13.1313\ldots\ldots) - (0.1313\ldots\ldots)$

$99x = 13.000$

Hence $x = \frac{13}{99}$

(iii) $0.\overline{67}$

Solution:

Let $x = 0.\overline{67}$

or $x = 0.676767\ldots\ldots$ .... **(i)**

Since we have only two digit i.e., 67 repeating indefinitely therefore multiplying both sides by 100

$100x = 67.676767\ldots\ldots$ .... **(ii)**

Subtracting (i) from (ii), we get

$100x - x = (67.6767\ldots\ldots) - (0.6767\ldots\ldots)$

$99x = 67.000$

Hence $x = \frac{67}{99}$

EXERCISE 2.2

Q1. Identify the property used in the following

(i)	$a + b = b + a$	**(ii)**	$(ab)c - a(bc)$
(iii)	$7 \times 1 = 7$	**(iv)**	$x > y$ or $x = y$ or $x < y$
(v)	$ab = ba$	**(vi)**	$a + c = b + c \Rightarrow a = b$
(vii)	$5 + (-5) = 0$	**(viii)**	$7 \times \frac{1}{7} = 1$
(ix)	$a > b \Rightarrow ac > bc \quad (c > 0)$		

Solution:

(i) Commutative Property w.r.t Addition
(ii) Associative Property w.r.t. Multiplication
(iii) Multiplicative Identity
(iv) Trichotomy Property
(v) Commutative Property w.r.t. Multiplication
(vi) Cancellation Property of Addition
(vii) Additive Inverse
(viii) Multiplicative Inverse
(ix) Multiplicative Property

Q2. Fill in the following blanks by stating the properties of real numbers used.

$3x + 3(y - x)$

$= 3x + 3y - 3x,$ ......

$= 3x - 3x + 3y,$ ......

$= 0 + 3y,$ ......

$= 3y,$ ......

Solution:

$3x + 3(y - x)$

Step 1:

$= 3x + 3y - 3x,$

Distributive Property w.r.t. Multiplication

Step 2:

$= 3x - 3x + 3y,$

Commutative Property w.r.t. Addition

Step 3:

$= 0 + 3y,$

Additive Inverse

Step 4:

$= 3y,$

Additive Identity

Q3. Give the name of property used in the following.

(i) $\sqrt{24} + 0 = \sqrt{24}$

(ii) $-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$

(iii) $\pi + (-\pi) = 0$

(iv) $\sqrt{3}\,.\sqrt{3}$ **is a real number**

(v) $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1$

Solution:

(i) Additive Identity

(ii) Distributive Property w.r.t. Multiplication

(iii) Additive Inverse

(iv) Closure Property

(v) Multiplicative Inverse

EXERCISE 2.3

Q1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

Solution:

(i) $\sqrt[3]{-64} = (-64)^{1/3}$

(ii) $2^{3/5} = (2^3)^{1/5} = \sqrt[5]{2^3}$

(iii) $-7^{1/3} = \sqrt[3]{-7} = -\sqrt[3]{7}$

(iv) $y^{-2/3} = (y^{-2})^{1/3} = \sqrt[3]{y^{-2}}$

Q2. Tell whether the following statements are true or false?

(i) $5^{1/5} = \sqrt{5}$ **(ii)** $2^{2/3} = \sqrt[3]{4}$

(iii) $\sqrt{49} = \sqrt{7}$ **(iv)** $\sqrt[3]{x^{27}} = x^3$

Solution:

(i) False **(ii)** True **(iii)** False **(iv)** False

Q3. Simplify the following radical expressions.

(i) $\sqrt[3]{-125}$

Solution:

$= \sqrt[3]{(-5)^3} = (-5)^{3\times\frac{1}{3}} = -5$

(ii) $\sqrt[4]{32}$

Solution:

$= \sqrt[4]{2^5} = \sqrt[4]{2.\ 2^4}$

$= \sqrt[4]{2}\,.\,\sqrt[4]{2^4} = \sqrt[4]{2}\,.\,(2)^{4\times\frac{1}{2}}$

$= \sqrt[4]{2}\,.2 = 2\sqrt[4]{2}$

(iii) $\sqrt[5]{\dfrac{3}{32}}$

Solution:

$= \dfrac{\sqrt[5]{3}}{\sqrt[5]{32}} = \dfrac{\sqrt[5]{3}}{\sqrt[5]{2^5}} = \dfrac{\sqrt[5]{3}}{2^{5\times\frac{1}{5}}}$

$= \dfrac{\sqrt[5]{3}}{2}$

(iv) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$$= \sqrt[3]{\left(-\frac{2}{3}\right)^3} = \left(-\frac{2}{3}\right)^{3\times\frac{1}{3}} = -\frac{2}{3}$$

EXERCISE 2.4

Q1. Use laws of exponents to simplify:

(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

Solution:

$$= \frac{(243)^{-2/3}(32)^{-1/5}}{(196)^{-\frac{1}{2}}}$$

$$= \left(\frac{1}{243}\right)^{2/3} \times \left(\frac{1}{32}\right)^{1/5} \times (196)^{1/2}$$

$$= \left(\frac{1}{3^5}\right)^{2/3} \times \left(\frac{1}{2^5}\right)^{1/5} \times (4\times 49)^{1/2}$$

$$= \frac{1}{3^{10/3}} \times \frac{1}{2^{5/5}} \times (2^2)^{1/2} \times (7^2)^{1/2}$$

$$= \frac{1}{3^{1/3}\times 3^{9/3}} \times \frac{1}{2} \times 2 \times 7 \qquad \left(\because \frac{1}{3^{10/3}} = \frac{1}{3^{1/3}\times 3^{9/3}}\right)$$

$$= \frac{1}{3^3\times 3^{1/3}} \times 7$$

$$= \frac{7}{3^3 \times \sqrt[3]{3}} = \frac{7}{27\left(\sqrt[3]{3}\right)}$$

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution:

$$= 2\times(-8)\times x^5 . x^{-3} . y^{-4} . y^2$$

$$= -16x^{5-3} . y^{-4+2} = -16x^2y^{-2}$$

$$= -\frac{16x^2}{y^2}$$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

Solution:

$$= \left(\frac{x^4y^{-3}z^0}{x^{-2}y^{-1}z^{-4}}\right)^3 = \left(\frac{x^{4+2} . z^{0+4}}{y^{3-1}}\right)^3$$

$$= \left(\frac{x^6 . z^4}{y^2}\right)^3 = \frac{x^{6\times 3} . z^{4\times 3}}{y^{2\times 3}}$$

(iv) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$$= \sqrt[3]{\left(-\frac{2}{3}\right)^3} = \left(-\frac{2}{3}\right)^{3\times\frac{1}{3}} = -\frac{2}{3}$$

EXERCISE 2.4

Q1. Use laws of exponents to simplify:

(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

Solution:

$$= \frac{(243)^{-2/3}(32)^{-1/5}}{(196)^{-\frac{1}{2}}}$$

$$= \left(\frac{1}{243}\right)^{2/3} \times \left(\frac{1}{32}\right)^{1/5} \times (196)^{1/2}$$

$$= \left(\frac{1}{3^5}\right)^{2/3} \times \left(\frac{1}{2^5}\right)^{1/5} \times (4\times 49)^{1/2}$$

$$= \frac{1}{3^{10/3}} \times \frac{1}{2^{5/5}} \times (2^2)^{1/2} \times (7^2)^{1/2}$$

$$= \frac{1}{3^{1/3}\times 3^{9/3}} \times \frac{1}{2} \times 2 \times 7 \qquad \left(\because \frac{1}{3^{10/3}} = \frac{1}{3^{1/3}\times 3^{9/3}}\right)$$

$$= \frac{1}{3^3\times 3^{1/3}} \times 7$$

$$= \frac{7}{3^3\times\sqrt[3]{3}} = \frac{7}{27\left(\sqrt[3]{3}\right)}$$

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution:

$$= 2\times(-8)\times x^5 . x^{-3} . y^{-4} . y^2$$

$$= -16x^{5-3} . y^{-4+2} = -16x^2y^{-2}$$

$$= -\frac{16x^2}{y^2}$$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

Solution:

$$= \left(\frac{x^4y^{-3}z^0}{x^{-2}y^{-1}z^{-4}}\right)^3 = \left(\frac{x^{4+2} . z^{0+4}}{y^{3-1}}\right)^3$$

$$= \left(\frac{x^6 . z^4}{y^2}\right)^3 = \frac{x^{6\times3} . z^{4\times3}}{y^{2\times3}}$$

$$= \frac{x^{18}\ z^{12}}{y^6}$$

(iv) $\dfrac{(81)^n \cdot 3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

Solution:

$$= \frac{(3^4)^n \cdot 3^5-(3)^{4n-1} \cdot 3^5}{(3^2)^{2n}\ (3^3)} = \frac{3^{4n} \cdot 3^5-3^{4n-1} \cdot 3^5}{3^{4n} \cdot 3^3}$$

$$= \frac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}} = \frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4}(3-1)}{3^{4n+3}} = 3^{4n+4-4n-3} \cdot (2)$$

$$= (3)\times(2) = 6$$

Q2. Show that $\left(\dfrac{x^a}{x^b}\right)^{a+b} \times \left(\dfrac{x^b}{x^c}\right)^{b+c} \times \left(\dfrac{x^c}{x^a}\right)^{c+a} = 1$

Solution:

$$= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$$

$$= (x^a.x^{-b})^{a+b} \times (x^b.x^{-c})^{b+c} \times (x^c.x^{-a})^{c+a}$$

$$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

Q3. Simplify

(i) $\dfrac{2^{1/3}\times(27)^{1/3}\times(60)^{1/2}}{(180)^{1/2}\times(4)^{-1/3}\times(9)^{1/4}}$

Solution:

$$= \frac{2^{1/3}\times(3^3)^{1/3}\times(3\ .\ 5\ .\ 2^2)^{1/2}}{(2\ .\ 2\ .\ 2\ .\ 3\ .\ 3\ .\ 5)^{1/2}\times(2^2)^{-1/3}\times(3^2)^{1/4}}$$

$$= \frac{2^{1/3}\ .\ 3^{3\times1/3}\ .\ 3^{1/2}\ .\ 5^{1/2}\ .\ 2^{2\times1/2}}{(2^2\ .\ 3^2\ .\ 5)^{1/2}\ .\ 2^{-2/3}\ .\ 3^{2/4}}$$

$$= \frac{2^{1/3}\ .\ 3\ .\ 3^{1/2}\ .\ 5^{1/2}\ .\ 2}{(2^2\ .\ 3^2\ .\ 5)^{1/2}\ .\ 2^{-2/3}\ .\ 3^{2/4}}$$

$$= \frac{2^{1/3}\ .\ 3^{1/2}\ .\ 5^{1/2}\ .\ 3\ .\ 2}{2\ .\ 3\ .\ 5^{1/2}\ .\ 2^{-2/3}\ .\ 3^{1/2}}$$

$$= \frac{2^{\frac{1}{3}+\frac{2}{3}}\ .\ 3^{\frac{1}{2}}\ .\ 5^{\frac{1}{2}}\ .\ 3\ .\ 2}{5^{\frac{1}{2}}\ \ 3^{\frac{1}{2}}\ \ 2\ \ 3}$$

$$= 2^{3/3}$$

$$= 2^1 = 2$$

(ii) $\sqrt{\dfrac{(216)^{2/3}\times(25)^{1/2}}{(0.04)^{-1/2}}}$

Solution:

$$= \sqrt{\dfrac{(2^3 \;.\; 3^3)^{2/3} \;.\; (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}}$$

$$= \sqrt{2^2 \quad 3^2 \quad 5 \quad \left(\frac{4}{100}\right)^{1/2}}$$

$$= \sqrt{2^2 \;.\; 3^2 \;.\; 5 \;.\; \left(\frac{2^2}{10^2}\right)^{1/2}}$$

$$= \sqrt{2^2 \;.\; 3^2 \;.\; 5 \;.\; \left(\frac{2}{10}\right)^{2\times\frac{1}{2}}}$$

$$= \sqrt{2^2 \quad 3^2 \;.\; 5 \;.\; \frac{2}{10}}$$

$$= \sqrt{2^2 \;.\; 3^2}$$

$$= 2 \quad 3$$

$$= 6$$

(iii) $5^2 \div (5^2)^3$

Solution:

$$= 5^8 \div 5^6 \quad = \quad \frac{5^8}{5^6} \quad = \quad 5^{8-6}$$

$$= 5^2 \quad = \quad 25$$

(iv) $(x^3)^2 \div x^{3^2}$, $x \neq 0$

Solution:

$$= x^{3\times2} \div x^{3\times3} \quad = \quad x^6 \div x^9 \quad = \quad \frac{x^6}{x^9}$$

$$= x^{6-9} \quad = \quad x^{-3} \quad = \quad \frac{1}{x^3}$$

EXERCISE 2.5

Q1. Evaluate

(i) i^7

Solution:

$$= i^6 \times i \quad = \quad (i^2)^3 \times i \quad (\because i^2 = -1)$$

$$= (-1)^3 \times i \quad = \quad (-1) \times -i \quad = \quad -i$$

(ii) i^{50}

(ii) $\sqrt{\dfrac{(216)^{2/3}\times(25)^{1/2}}{(0.04)^{-1/2}}}$

Solution:

$$= \sqrt{\frac{(2^3 \cdot 3^3)^{2/3} \cdot (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}}$$

$$= \sqrt{2^2 \quad 3^2 \quad 5 \quad \left(\frac{4}{100}\right)^{1/2}}$$

$$= \sqrt{2^2 \cdot 3^2 \cdot 5 \cdot \left(\frac{2^2}{10^2}\right)^{1/2}}$$

$$= \sqrt{2^2 \cdot 3^2 \cdot 5 \cdot \left(\frac{2}{10}\right)^{2\times\frac{1}{2}}}$$

$$= \sqrt{2^2 \quad 3^2 \cdot 5 \cdot \frac{2}{10}}$$

$$= \sqrt{2^2 \cdot 3^2}$$

$$= 2 \quad 3$$

$$= 6$$

(iii) $5^2 \div (5^2)^3$

Solution:

$$= 5^8 \div 5^6 = \frac{5^8}{5^6} = 5^{8-6}$$

$$= 5^2 = 25$$

(iv) $(x^3)^2 \div x^{3^2}$, $x \neq 0$

Solution:

$$= x^{3\times2} \div x^{3\times3} = x^6 \div x^9 = \frac{x^6}{x^9}$$

$$= x^{6-9} = x^{-3} = \frac{1}{x^3}$$

EXERCISE 2.5

Q1. Evaluate

(i) i^7

Solution:

$$= i^6 \times i = (i^2)^3 \times i \qquad (\because i^2 = -1)$$

$$= (-1)^3 \times i = (-1) \times -i = -i$$

(ii) i^{50}

Solution:

$= \quad (i^2)^{25} \quad = \quad (-1)^{25} = \quad -1 \quad (\because i^2 = -1)$

(iii) i^{12}

Solution:

$= \quad (i^2)^6 \quad = \quad (-1)^6 \quad = \quad 1 \quad (\because i^2 = -1)$

(iv) $(-i)^8$

Solution:

$= \quad i^8 \quad = \quad (i^2)^4 \quad (\because i^2 = -1)$

$= \quad (-1)^4 \quad = \quad 1$

(v) $(-i)^5$

Solution:

$= \quad -i^5 \quad = \quad -i^4 \times i$

$= \quad -(i^2)^2 \times i \quad = \quad -i \quad (\because i^2 = -1)$

(vi) i^{27}

Solution:

$= \quad i^{26} \times i \quad = \quad (i^2)^{23} \times i$

$= \quad (-1)^{23} \times i \quad = \quad (-1) \times i$

$= \quad -i \quad (\because i^2 = -1)$

Q2. Write the conjugate of the following numbers.

(i) $2 + 3i$

Solution:

Let $z = 2 + 3i$

Then $\bar{z} = 2 - 3i$

(ii) $3 - 5i$

Solution:

Let $z = 3 - 5i$

Then $\bar{z} = 3 + 5i$

(iii) $-i$

Let $z = -i$

Then $\bar{z} = i$

(iv) $-3 + 4i$

Solution:

Let $z = -3 + 4i$

Then $\bar{z} = -3 - 4i$

(v) $-4 - i$

Solution:

Let $z = -4 - i$

Then $\bar{z} = -4 + i$

(vi) $i - 3$

Solution:

Let $z = i - 3$

Then $\bar{z} = -i - 3$

Q3. Write the real and imaginary part of the following numbers.

Solution:

(i)	$1 + i$	Re(z)	=	1	Im(z)	=	1
(ii)	$-1 + 2i$	Re(z)	=	-1	Im(z)	=	2
(iii)	$-3i + 2$	Re(z)	=	2	Im(z)	=	-3
(iv)	$-2 - 2i$	Re(z)	=	-2	Im(z)	=	-2
(v)	$-3i$	Re(z)	=	0	Im(z)	=	-3
(vi)	$2 + 0i$	Re(z)	=	2	Im(z)	=	0

Q4. Find the value of x and y if $x + iy + 1 = 4 - 3i$.

Solution:

$$x + iy + 1 = 4 - 3i$$

$$(x + 1) + iy = 4 - 3i$$

By comparing real and imaginary parts, we get

$x + 1 = 4$ and $y = -3$

$x = 4 - 1$ and $y = -3$

$x = 3$ and $y = -3$

EXERCISE 2.6

Q1. Identify the following statements as true or false.

(i) $\sqrt{-3}\sqrt{-3} = 3$

(ii) $i^{73} = -i$

(iii) $i^{10} = -i$

(iv) **Complex conjugate of** $(-6i + i^2)$ **is** $(-1 + 6i)$

(v) **Difference of a complex number $z = a + bi$ and its conjugate is a real number.**

(vi) **If** $(a - 1) - (b + 3)i = 5 + 8i$ **, then** $a = 6$ **and** $b = -11$

(vii) **Product of a complex number and its conjugate is always a non-negative real number.**

Solution:

(i)	False	**(ii)**	False	**(iii)**	True	**(iv)**	True
(v)	False	**(vi)**	True	**(vii)**	True		

Solution:

Let $z = i - 3$

Then $\bar{z} = -i - 3$

Q3. Write the real and imaginary part of the following numbers.

Solution:

(i)	$1 + i$	$\text{Re}(z) = 1$	$\text{Im}(z) = 1$
(ii)	$-1 + 2i$	$\text{Re}(z) = -1$	$\text{Im}(z) = 2$
(iii)	$-3i + 2$	$\text{Re}(z) = 2$	$\text{Im}(z) = -3$
(iv)	$-2 - 2i$	$\text{Re}(z) = -2$	$\text{Im}(z) = -2$
(v)	$-3i$	$\text{Re}(z) = 0$	$\text{Im}(z) = -3$
(vi)	$2 + 0i$	$\text{Re}(z) = 2$	$\text{Im}(z) = 0$

Q4. Find the value of x and y if $x + iy + 1 = 4 - 3i$.

Solution:

$$x + iy + 1 = 4 - 3i$$
$$(x + 1) + iy = 4 - 3i$$

By comparing real and imaginary parts, we get

$x + 1 = 4$ and $y = -3$

$x = 4 - 1$ and $y = -3$

$x = 3$ and $y = -3$

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(vii) **Product of a complex number and its conjugate is always a non-negative real number.**

Solution:

(i) False **(ii)** False **(iii)** True **(iv)** True

(v) False **(vi)** True **(vii)** True

Q2. Express each complex number in the standard form $a + bi$, where a and b are real numbers.

Solution:

(i) $(2 + 3i) + (7 - 2i)$

Solution:

By separating real and imaginary parts, we get

$= (2 + 7) + (3 - 2)i = 9 + i$

(ii) $2(5 + 4i) - 3(7 + 4i)$

Solution:

By separating real and imaginary parts, we get

$= 10 + 8i - 21 - 12i = -11 - 4i$

(iii) $-(-3 + 5i) - (4 + 9i)$

Solution:

By separating real and imaginary parts, we get

$= 3 - 5i - 4 - 9i = -1 - 14i$

(iv) $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

Solution:

By separating real and imaginary parts, we get

$= 2(-1) + 6i \cdot i^2 + 3(i^2)^8 - 6i^{18} \cdot i + 4i^{24} \cdot i$

$= -2 + 6i(-1) + 3(1) - 6(i^2)^9 i + 4(i^2)^{22} i$

$= -2 - 6i + 3 - 6i(-1) + 4i \quad (\because i^2 = -1)$

$= -2 - 6i + 3 + 6i + 4i$

$= 1 + 4i$

Q3. Simplify and write your answer in the form $a + bi$.

Solution:

(i) $(-7 + 3i)(-3 + 2i)$

Solution:

$= 21 - 14i - 9i + 6i^2$

$= 21 - 23i + 6(-1) \quad (\because i^2 = -1)$

$= 21 - 23i - 6$

$= 15 - 23i$

(ii) $(2 - \sqrt{-4})(3 - \sqrt{-4})$

Solution:

$= (2 - 2i)(3 - 2i)$

$= 6 - 4i - 6i + 4i^2$

$= 6 - 10i + 4(-1)$

$= 6 - 10i - 4 \quad (\because i^2 = -1)$

$= 2 - 10i$

(iii) $\left(\sqrt{5}-3i\right)^2$

Solution:

$= \left(\sqrt{5}\right)^2 + (3i)^2 - 2\left(\sqrt{5}\right)(3i)$

$= 5 + 9i^2 - 6\sqrt{5}i$

$= 5 + 9(-1) - 6\sqrt{5}i \qquad (\because i^2 = -1)$

$= 5 - 9 - 6\sqrt{5}i$

$= -4 - 6\sqrt{5}i$

(iv) $(2-3i)\overline{(3-2i)}$

Solution:

$= (2-3i)(3+2i) \qquad = \qquad 6 + 4i - 9i - 6i^2$

$= 6 - 5i - 6(-1) \qquad (\because i^2 = -1)$

$= 6 + 6 - 5i$

$= 12 - 5i$

Q4. Simplify and write your answer in the form $a + bi$.

(i) $\frac{-2}{1+i}$

Solution:

$= \frac{-2}{1+i} \times \frac{1-i}{1-i} \qquad = \qquad \frac{-2(1-i)}{1-i^2}$

$= \frac{-2+2i}{1+1} \qquad = \qquad \frac{-2+2i}{2} \qquad (\because i^2 = -1)$

$= -1 + i$

(ii) $\frac{2+3i}{4-i}$

Solution:

$= \frac{2+3i}{4-i} \times \frac{4+i}{4+i} \qquad = \qquad \frac{(2+3i)(4+i)}{16-i^2}$

$= \frac{8+2i+12i+3i^2}{16+1} \qquad = \qquad \frac{8+14i-3}{17} \qquad (\because i^2 = -1)$

$= \frac{5+14i}{17} \qquad = \qquad \frac{5}{17} + \frac{14}{17}i$

(iii) $\frac{9-7i}{3+i}$

Solution:

$= \frac{9-7i}{3+i} \times \frac{3-i}{3-i} \qquad = \qquad \frac{(9-7i)(3-i)}{9-i^2}$

$= 27-9i-21i+7i^2 \qquad = \qquad \frac{27-30i-7}{10} \qquad (\because i^2 = -1)$

$= \qquad 10 \qquad = \qquad \frac{20}{10} - \frac{30}{10}i$

$= 2 - 3i$

(iv) $\frac{2-6i}{3+i}-\frac{4+i}{3+i}$

Solution:

$$= \frac{2-6i-4-i}{3+i} = \frac{-2-7i}{3+i}$$
$$= \frac{-2-7i}{3+i}\times\frac{3-i}{3-i} = \frac{(-2-7i)(3-i)}{9-i^2}$$
$$= \frac{-6+2i-21i+7i^2}{9+1} = \frac{-6-19i-7}{10} \qquad (\because i^2=-1)$$
$$= \frac{-13-19i}{10} = -\frac{13}{10}-\frac{19}{10}i$$

(v) $\left(\frac{1+i}{1-i}\right)^2$

Solution:

$$= \left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^2 = \left(\frac{(1+i)^2}{1-i^2}\right)^2$$
$$= \left(\frac{1+2i+i^2}{1+1}\right)^2 = \left(\frac{1+2i-1}{2}\right)^2 \qquad (\because i^2=-1)$$
$$= \left(\frac{2i}{2}\right)^2 = i^2$$
$$= -1 \qquad (\because i^2=-1)$$

(vi) $\frac{1}{(2+3i)(1-i)}$

Solution:

$$= \frac{1}{2-2i+3i-3i^2} = \frac{1}{2+i-3(-1)}$$
$$= \frac{1}{2+3+i} = \frac{1}{5+i}$$
$$= \frac{1}{5+i}\times\frac{5-i}{5-i} = \frac{5-i}{25-i^2}$$
$$= \frac{5-i}{25+1} = \frac{5-i}{26}$$
$$= \frac{5}{26}-\frac{1}{26}i$$

Q5. Calculate (a) $\bar{z}$ (b) $z+\bar{z}$ (c) $z-\bar{z}$ (d) $z\bar{z}$ for each of the following

(i) $z=-i$ **(ii)** $z=2+i$

(iii) $z=\frac{1+i}{1-i}$ **(iv)** $z=\frac{4-3i}{2+4i}$

Solution:

(i) $z=-i$

$z=0-i$

(a) $\bar{z}=0+i=i$

(b) $z+\bar{z}=-i+i=0$
(c) $z-\bar{z}=-i-i=-2i$
(d) $z\bar{z}=(-i)(i)=-i^2=-(-1)=1 \qquad (\because i^2=-1)$
(ii) $z=2+i$
(a) $\bar{z}=2-i$
(b) $z+\bar{z}=2+i+2-i=4$
(c) $z-\bar{z}=2+i-2+i=2i$
(d) $z\bar{z}=(2+i)(2-i)=4-i^2=4-(-1)=4+1=5$
(iii) $z=\frac{1+i}{1-i}$

$$z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{1-i^2}=\frac{1+2i+i^2}{1-(-1)}=\frac{1+2i-1}{1+1}=\frac{2i}{2}=0+i$$

(a) $\bar{z}=0-i=-i$
(b) $z+\bar{z}=i-i=0$
(c) $z-\bar{z}=i-(-i)=i+i=2i$
(d) $z\bar{z}=(i)(-i)=-i^2=-(-1)=1$
(iv) $z=\frac{4-3i}{2+4i}$

$$z=\frac{4-3i}{2+4i}\times\frac{2-4i}{2-4i}=\frac{(4-3i)(2-4i)}{4-16i^2}=\frac{8-16i-6i+12i^2}{4-16(-1)}$$
$$=\frac{8-22i+12(-1)}{4+16}=\frac{8-22i-12}{20}=\frac{-4-22i}{20}=-\frac{1}{5}-\frac{11}{10}i$$

(a) $\bar{z}=-\frac{1}{5}+\frac{11}{10}i$
(b) $z+\bar{z}=-\frac{1}{5}-\frac{11}{10}i-\frac{1}{5}+\frac{11}{10}i=-\frac{2}{5}$
(c) $z-\bar{z}=-\frac{1}{5}-\frac{11}{10}i+\frac{1}{5}-\frac{11}{10}i=-\frac{22}{10}i=-\frac{11}{5}i$
(d) $z\bar{z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)\left(-\frac{1}{5}+\frac{11}{10}i\right)=\left(-\frac{1}{5}\right)^2-\left(\frac{11}{10}i\right)^2$

$$=\frac{1}{25}-\frac{121}{100}i^2=\frac{1}{25}+\frac{121}{100}=\frac{4+121}{100}=\frac{125}{100}=\frac{5}{4}$$

Q6. If $z=2+3i$ and $w=5-4i$, show that

(i) $\overline{z+w}=\bar{z}+\bar{w}$ **(ii)** $\overline{z-w}=\bar{z}-\bar{w}$

(iii) $\overline{zw}=\bar{z}\,\bar{w}$

(iv) $\overline{\left(\frac{z}{w}\right)}=\frac{\bar{z}}{\bar{w}}$, **where $w\neq 0$.**

(v) $\frac{1}{2}(z+\bar{z})$ **is the real part of z.**

(vi) $\frac{1}{2i}(z-\bar{z})$ **is the imaginary part of z.**

Solution:

$z=2+3i \quad\Rightarrow\quad \bar{z}=2-3i$

$w=5-4i \quad\Rightarrow\quad \bar{w}=5+4i$

(i) $\overline{z+w} = \bar{z} + \bar{w}$

$z + w = 2 + 3i + 5 - 4i = 7 - i$

L.H.S. = $\overline{z+w} = 7 + i$

R.H.S. = $\bar{z} + \bar{w} = 2 - 3i + 5 + 4i = 7 + i$

Hence L.H.S. = R.H.S.

(ii) $\overline{z-w} = \bar{z} - \bar{w}$

$z - w = 2 + 3i - 5 + 4i = -3 + 7i$

L.H.S. = $\overline{z-w} = -3 - 7i$

R.H.S. = $\bar{z} - \bar{w} = 2 - 3i - 5 - 4i = -3 - 7i$

Hence L.H.S. = R.H.S.

(iii) $\overline{zw} = \bar{z}\,\bar{w}$

$zw = (2 + 3i)(5 - 4i) = 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$ $\quad (\because i^2 = -1)$

$= 10 + 12 + 7i = 22 + 7i$

L.H.S. = $\overline{zw} = 22 - 7i$

R.H.S. = $\bar{z}\,\bar{w} = (2 - 3i)(5 + 4i) = 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1) = 10 + 12 - 7i = 22 - 7i$

Hence L.H.S. = R.H.S.

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$**, where** $w \neq 0$**.**

$$\frac{z}{w} = \frac{2+3i}{5-4i} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i} = \frac{(2+3i)(5+4i)}{25-16i^2} = \frac{10+8i+15i+12i^2}{25-16(-1)}$$

$$= \frac{10+23i-12}{25+16} = \frac{-2+23i}{41} = -\frac{2}{41} + \frac{23}{41}i \qquad (\because i^2 = -1)$$

L.H.S. = $\overline{\left(\frac{z}{w}\right)} = -\frac{2}{41} - \frac{23}{41}i$

R.H.S. = $\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i} = \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i} = \frac{(2-3i)(5-4i)}{25-16i^2}$

$$= \frac{10-8i-15i+12i^2}{25-16(-1)} = \frac{10-12-23i}{25+16}$$

$$= \frac{-2-23i}{41} = -\frac{2}{41} - \frac{23}{41}i$$

Hence L.H.S. = R.H.S.

(v) $\frac{1}{2}(z + \bar{z})$ **is the real part of z.**

$\frac{1}{2}(z + \bar{z}) = \frac{1}{2}(2 + 3i + 2 - 3i)$

$= \frac{1}{2}(4) = 2$ is real part of z

(vi) $\frac{1}{2i}(z - \bar{z})$ **is the imaginary part of z.**

$\frac{1}{2i}(z - \bar{z}) = \frac{1}{2i}(2 + 3i - 2 + 3i)$

$= \frac{1}{2i}(6i) = 3$ is imaginary part of z

Q7. Solve the following equations for real x and y.

(i) $(2-3i)(x+yi) = 4+i$

(ii) $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

(iii) $(3+4i)^2 - 2(x-yi) = x+yi$

(i) $(2-3i)(x+yi) = 4+i$

Solution:

$2x + 2yi - 3xi - 3yi^2 = 4 + i$

$2x - 3y(-1) - 3xi + 2yi = 4 + i$

$(2x+3y) + (2y-3x)i = 4 + i$

By comparing real and imaginary parts, we get

$2x + 3y = 4$ (i)

$2y - 3x = 1$

or $-3x + 2y = 1$ .... (ii)

Now multiplying eq. (i) by 3 and eq. (ii) by 2

$6x + 9y = 12$ .... (iii)

$-6x + 4y = 2$ .... (iv)

Adding eq. (iii) and eq. (iv)

$$\begin{array}{r} 6x + 9y = 12 \\ \underline{-6x + 4y = 2} \\ 13y = 14 \end{array} \quad \Rightarrow \quad y = \frac{14}{13}$$

Put $y = \frac{14}{13}$ in eq. (i)

$2x + 3y = 4$

$2x + 3\left(\frac{14}{13}\right) = 4$

$2x + \frac{42}{13} = 4$

$2x = 4 - \frac{42}{13} = \frac{52-42}{13} = \frac{10}{13}$

$x = \frac{10}{13} \times \frac{1}{2} = \frac{5}{13}$

Hence $x = \frac{5}{13}$ and $y = \frac{14}{13}$

(ii) $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

Solution:

$3x + 3yi - 2xi - 2yi^2 = 2x - 4yi + 2i - 1$

$3x - 2y(-1) - 3yi + 2xi = 2x - 1 - 4yi + 2i$

$(3x+2y) + (3y-2x)i = (2x-1) + (2-4y)i$

By comparing real and imaginary parts, we get

$3x + 2y = 2x - 1$

or $3x - 2x + 2y = -1$

or $x + 2y = -1$ .... **(i)**

$3y - 2x = 2 - 4y$

or $3y - 2x + 4y = 2$

or $7y - 2x = 2$ .... **(ii)**

Multiplying eq. (i) by 2 and add in eq. (ii)

$$\begin{array}{r} 2x + 4y = -2 \\ \underline{-2x + 7y = 2} \\ 11y = 0 \end{array} \quad \Rightarrow \quad y = 0$$

Put $y = 0$ in eq. (i)

$x + 2y = -1$

$x + 2(0) = -1 \quad \Rightarrow \quad x = -1$

Hence $x = -1$ and $y = 0$

(iii) $(3 + 4i)^2 - 2(x - yi) = x + yi$

Solution:

$9 + 16i^2 + 24i - 2x + 2yi = x + yi$

$9 - 16 + 24i - 2x + 2yi = x + yi \quad (\because i^2 = -1)$

$(-7 - 2x) + (24 + 2y)i = x + yi$

By comparing real and imaginary parts, we get

$-7 - 2x = x$

$-2x - x = 7 \quad \Rightarrow \quad -3x = 7$

$\Rightarrow \quad x = -\frac{7}{3}$

$24 + 2y = y$

or $2y - y = -24$

$y = -24$

Hence $x = -\frac{7}{3}$ and $y = -24$

REVIEW EXERCISE 2

Q1. Multiple Choice Questions. Choose the correct answer.

(i) $(27x^{-1})^{-2/3}$.........

(a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$ (c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $\frac{\sqrt{x^3}}{8}$

(ii) **Write $\sqrt[7]{x}$ in exponential form........**

(a) x (b) x^7 (c) $x^{1/7}$ (d) $x^{7/2}$

(iii) **Write $4^{2/3}$ with radical sign.........**

(a) $\sqrt[3]{4^2}$ (b) $\sqrt{4^3}$ (c) $\sqrt[2]{4^3}$ (d) $\sqrt{4^6}$